THE SCOTS COLLEGE

YEAR 12

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

EXTENSION 1

AUGUST 2004

TIME ALLOWED: **2 HOURS** *[plus 5 minutes reading time]*

INSTRUCTIONS:

- Attempt all questions.
- Write using a blue or black pen.
- Board approved calculators may be used.
- Show all necessary working.
- Diagrams are <u>NOT</u> to scale.

STUDENTS ARE ADVISED THAT THIS IS A TRIAL EXAMINATION ONLY AND CANNOT IN ANY WAY GUARANTEE THE CONTENT OR THE FORMAT OF THE HIGHER SCHOOL CERTIFICATE EXAMINATION.

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QUESTION 1

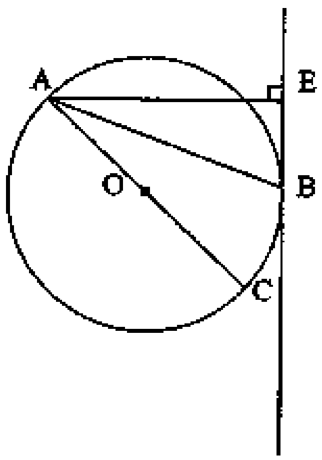
(a) Simplify $\dfrac{xy^{-1} - yx^{-1}}{x-y}$ [2]

(b) Find the coordinates of the point P which divides the interval AB externally in the ratio 5 : 2, given A(−5, 12) and B(4, 9). [2]

(c) Express $f(x) = x^3 + 3x^2 - 10x - 24$ as a product of three linear factors. [3]

(d) Evaluate $\int_0^{\pi/6} \sin^2 2x \, dx$, leaving your answer in exact form. [3]

(e) Two points A and B are on the circumference of a circle and AC is a diameter. AE is perpendicular to the tangent at B. Prove AB bisects ∠CAE. [2]



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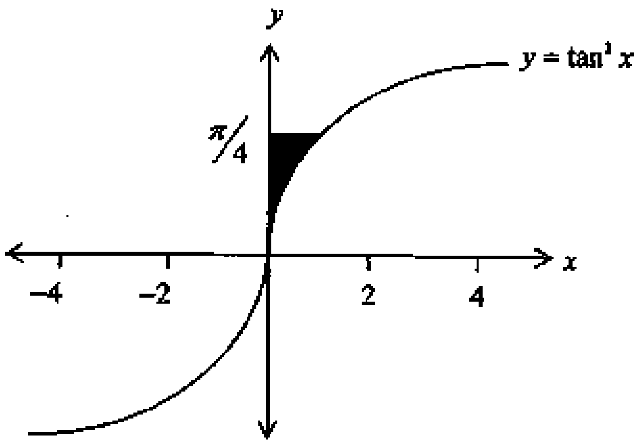
QUESTION 2

(a) Find the exact value of $\int_1^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}}\,dx$ [2]

(b) Evaluate $\int_0^1 x\sqrt{1-x^2}\,dx$, using the substitution $u = 1 - x^2$ [3]

(c) The shaded region in the diagram is bounded by the curve $y = \tan^{-1} x$, the line $y = \frac{\pi}{4}$ and the y-axis. Show that if the shaded region is rotated about the y-axis, then the volume generated is $\frac{\pi(4-\pi)}{4}$ units^3. [3]



(d) Use the principle of mathematical induction to show that $2^{3n} - 1$ is divisible by 7, for all $n \geq 0$. [4]

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QUESTION 3

(a) Express $3\cos x + 4\sin x$ in the form $A\cos(x-\alpha)$, where $A > 0$. Hence, or otherwise, solve $3\cos x + 4\sin x = -3$ for $0 \le x \le 360^\circ$. [4]

(b) Find the greatest coefficient in the expansion of $(3+4x)^{16}$, leaving your answer in index form. [4]

(c) The velocity v m/s of a particle moving along the x axis is given by $v^2 = 16x - 4x^2 + 20$.

(i) Prove that the motion is simple harmonic.

(ii) Find the centre of motion.

(iii) Find the length travelled by the particle in one oscillation. [4]

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QUESTION 4

(a) α, β and γ are the roots of the equation $2x^3 + 3x^2 - 4 = 0$. Find:

(i) $\alpha + \beta + \gamma$

(ii) $\alpha\beta\gamma$

(iii) $\alpha^2 + \beta^2 + \gamma^2$ [3]

(b) Given the function $y = 3\cos^{-1}\left(\frac{x}{2}\right)$:

(i) write down the domain and range;

(ii) sketch the graph of the function. [3]

(c) The graph of $y = x^3 + x - 1$ has a root close to $x = 0.5$. Find a better approximation to this root using one application of Newton's method. [3]

(d) Prove $\dfrac{2}{\tan A + \cot A} = \sin 2A$ [3]

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QUESTION 5

(a) Find the acute angle between the lines $2x-y+5=0$ and $y=-3x+7$. **[3]**

(b) In a school, a group of 5 students starts a rumour. The number, N, of students who have heard the rumour after t days is given by $N=A(1+e^{-kt})$, where A and k are constants. After three days, 80 students have heard the rumour and eventually all 560 students in the school have heard the rumour.

(i) Find the value of A.

(ii) Find the value of k.

(iii) Find within how many days it takes for all 560 students to have heard the rumour. **[5]**

(c) A pupil investigated a differentiable function $f(x)$ and found the following information:

$f(x)$ has its only zero at $x=-1$, $f(0)=2$, $\lim_{x \to \infty} f(x)=0$

(i) Draw a graph of the possible shape of $f(x)$.

(ii) Use your graph to demonstrate that $f(x)$ must have an inflexion point to the right of $x=-1$.

[4]

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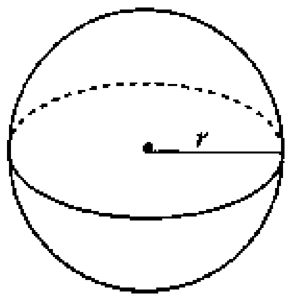
QUESTION 6

(a)



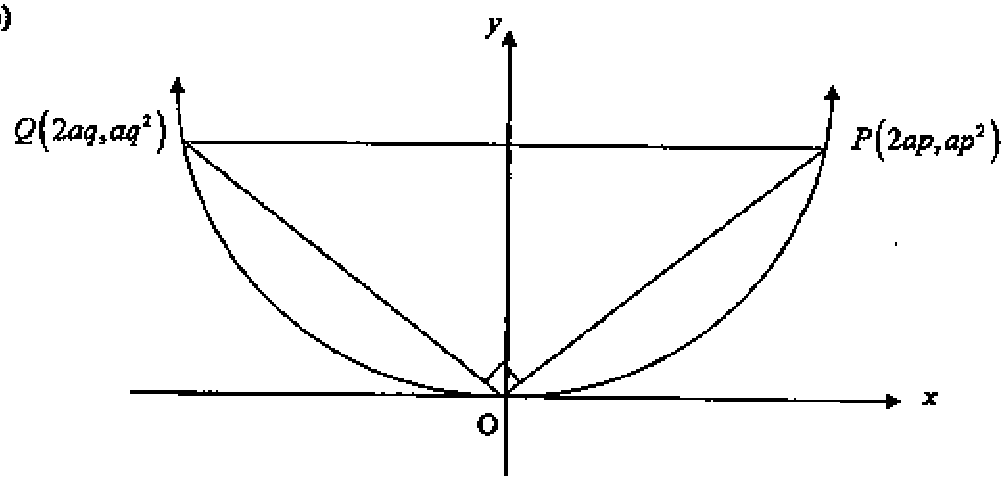
A spherical balloon is being inflated and its volume increases at a constant rate of 50mm^3 per second. At what rate is its surface area increasing when the radius is 20mm?

[5]

$V = \frac{4}{3}\pi r^3$

$A = 4\pi r^2$

(b)



PQ is a variable chord of the parabola $x^2 = 4ay$.

It subtends a right angle at the vertex O.

If p and q are the parameters corresponding to the points P, Q respectively:

(i) Show that the equation of the tangent to $x^2 = 4ay$ at P is $y - px + ap^2 = 0$.

(ii) Hence write down the equation of the tangent at Q, and then find R, the point of intersection of the two tangents drawn from P and Q.

(iii) Find the gradients of PO and QO and hence prove $pq = -4$.

(iv) Show that the locus of this point of intersection is $y = -4a$.

[7]

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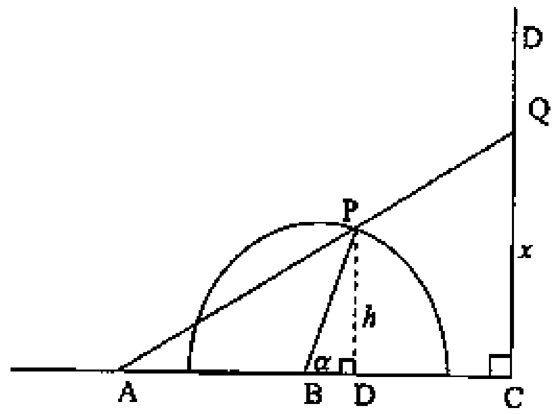
QUESTION 7

(a) In the figure, ABC is a straight line with $AB = BC = 3$. CD is perpendicular to ABC. On the semicircle with centre B and radius 2 is a variable point, P, with $\angle CBP = \alpha$ radians. The perpendicular from P to AC has length h. The line AP is produced to meet CD at Q and $QC = x$.

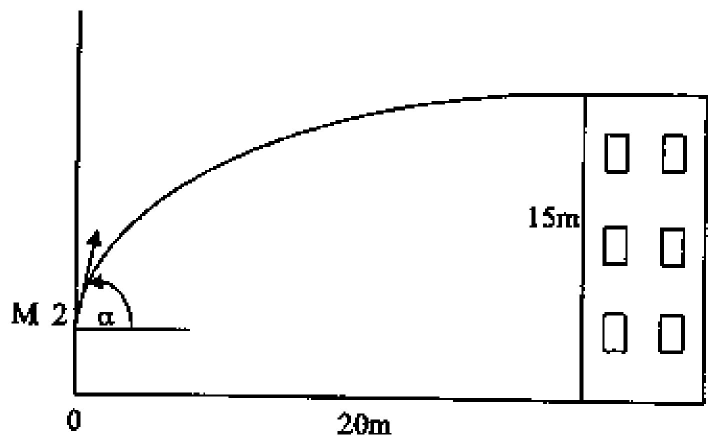


(i) Find an expression for h in terms of α.

(ii) Using similar triangles or otherwise, show that $x = \dfrac{12\sin\alpha}{3+2\cos\alpha}$.

(iii) Using calculus, find the maximum value of x, leaving your answer in exact form. [7]

(b) A man of height 2 metres throws a ball from M to the roof of a 15 metre high building. He throws the ball at an initial velocity of 25m/s, and he is 20m from the base of the building.



Between which two angles of projection (to the nearest degree) must he throw the ball to ensure that it lands on the roof of the building?

(Assume $\ddot{x} = 0$ and $\ddot{y} = -10$)

[5]

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1. (a) $\dfrac{xy^{-1}-yx^{-1}}{x-y} = \dfrac{\frac{x}{y}-\frac{y}{x}}{x-y}$

$= \dfrac{x^2-y^2}{xy(x-y)}$

$= \dfrac{x+y}{xy}$ ②

(b) External division, so let ratio be $-5:2$

A$(-5,12)$ B$(4,9)$

$= \dfrac{-5\times 2+4\times(-5)}{-5+2}$ $\quad y = \dfrac{12\times 2+9\times(-5)}{-5+2}$

$= 10$ $\quad = 7$

$\therefore$ P: $(10,7)$ ②

(c) $f(x) = x^3+3x^2-10x-24$

$f(1) = 1+3-10-24 \neq 0$

$f(-2) = -8+12+20-24 = 0$

$\therefore (x+2)$ is a factor

$$
\begin{array}{r}
x^2 + x - 12 \\
x+2 \overline{\big) x^3+3x^2-10x-24} \\
x^3+2x^2 \qquad\qquad \\
\hline
x^2-10x \qquad \\
x^2+2x \qquad \\
\hline
-12x-24 \\
-12x-24 \\
\hline
0
\end{array}
$$

$\therefore f(x) = (x+2)(x^3+x-12)$

$= (x+2)(x+4)(x-3)$. ③

(d) $\displaystyle\int_0^{\pi/6} \sin^2 2x\,dx = \frac{1}{2}\int_0^{\pi/6}(1-\cos 4x)\,dx$

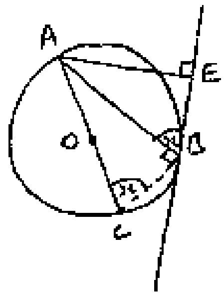
$\left[\begin{array}{l}\cos 2x = 1-2\sin^2 x \\ \sin^2 x = \frac{1}{2}(1-\cos 2x)\end{array}\right]$ $\quad = \frac{1}{2}\left[x-\frac{1}{4}\sin 4x\right]_0^{\pi/6}$

$= \frac{1}{2}\left(\left[\frac{\pi}{6}-\frac{1}{4}\sin\frac{4\pi}{6}\right]-[0]\right)$

$= \frac{1}{2}\left(\frac{\pi}{6}-\frac{1}{4}\times\frac{\sqrt{3}}{2}\right)$

$= \frac{1}{2}\left(\frac{\pi}{6}-\frac{\sqrt{3}}{8}\right)$ ③

(e)



$\angle ABE = \angle ACB = x$ (angle in alt. segment equal)

$\angle ABC = 90°$ ($\angle$ in semi-circle)

$\therefore \angle CAB = 90 - x$ ($\angle$ sum of Δ)

and $\angle BAE = 90 - x$ ($\angle$ sum of Δ)

$\therefore \angle CAB = \angle BAE$ ②

$\therefore$ AB bisects $\angle CAE$.

(Alternative proofs are possible!)

2. (a) $\displaystyle\int_1^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}}\,dx = \left[\sin^{-1}\frac{x}{2}\right]_1^{\sqrt{3}}$

$= \left[\sin^{-1}\frac{\sqrt{3}}{2}\right]-\left[\sin^{-1}\frac{1}{2}\right]$

$= \frac{\pi}{3}-\frac{\pi}{6} = \frac{\pi}{6}$. ②

(b) $\displaystyle\int_0^1 x\sqrt{1-x^2}\,dx = \int_1^0 u^{\frac{1}{2}}\cdot\frac{du}{-2}$

Let $u = 1-x^2$
$\frac{du}{dx} = -2x$
$\frac{du}{-2} = x.dx$
$x=1,\ u=0$
$x=0,\ u=1$

$= \displaystyle\int_0^1 \frac{1}{2}u^{\frac{1}{2}}\,du$

$= \left[\frac{1}{3}u^{\frac{3}{2}}\right]_0^1$

$= \left[\frac{1}{3}\right]-[0]$

$= \frac{1}{3}$ ③

2.(c) $y = \tan^{-1} x \qquad x = \tan y$

$x^2 = \tan^2 y$

$$\text{Vol} = \pi \int_0^{\pi/4} \tan^2 y \, dy$$

$$= \pi \int_0^{\pi/4} (\sec^2 y - 1) \, dy$$

$$= \pi \left[\tan y - y \right]_0^{\pi/4}$$

$$= \pi \left(\left[\tan \pi/4 - \pi/4 \right] - [0] \right)$$

$$= \pi \left(1 - \pi/4 \right) \qquad (3)$$

$$= \pi/4 \, (4 - \pi) = \frac{\pi(4-\pi)}{4} \text{ units}^3$$

(as reqd)

(d) Prove that $2^{3n} - 1$ is div. by 7.

For $n=1$, $2^3 - 1 = 7$ which is div by 7, ∴ true for $n=1$.

Assume true for $n=k$, ie.

$2^{3k} - 1 = 7M$ (*), where M is an integer, $M > 0$.

If true for $n=k$, show true for $n=k+1$, ie. show that $2^{3(k+1)} - 1$ is div. by 7:

$$2^{3k+3} - 1 = 2^{3k} \cdot 2^3 - 1$$

$$= (7M+1) \cdot 8 - 1$$

(from (*))

$$= 56M + 7$$

$$= 7(8M+1),$$

$M > 0$ is an integer, ∴ $8M+1$ is an integer, ∴ $2^{3(k+1)} - 1$ is div. by 7.

Thus if true for $n=k$, it is true for $n=k+1$.

It is true for $n=1$, ∴ by the principle of mathematical induction, it is true for all n. (4)

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3. (a) $3\cos x + 4\sin x \equiv A\cos(x-\alpha)$

$\text{RHS} = A\cos(x-\alpha)$

$= A\cos x \cos\alpha + A\sin x \sin\alpha$

Equating coeffs:

$A\cos\alpha = 3 \quad (1)$

$A\sin\alpha = 4 \quad (2)$

$\frac{(2)}{(1)}: \quad \tan\alpha = \frac{4}{3} \qquad \alpha = 53°8'$

$(1)^2 + (2)^2$:

$A^2\cos^2\alpha + A^2\sin^2\alpha = 9 + 16$

$A^2 = 25$

∴ $A = 5$

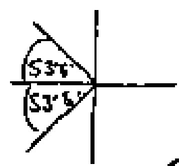
∴ $3\cos x + 4\sin x = 5\cos(x - 53°8')$

Solve $5\cos(x - 53°8') = -3$

$\cos(x - 53°8') = -3/5$

$x - 53°8' = 126°52'$

or $233°8'$



$x = 180°$ or $286°16'$ (4)

(b) $(3+4x)^{16}$

$T_{k+1} = {}^{16}C_k \cdot 3^{16-k} \cdot (4x)^k$

$T_k = {}^{16}C_{k-1} \cdot 3^{16-(k-1)} \cdot (4x)^{k-1}$

$= {}^{16}C_{k-1} \cdot 3^{17-k} \cdot (4x)^{k-1}$

3(b) (cont.)

ratio of coeffs:

$$\frac{T_{k+1}}{T_k} = \frac{{}^{16}C_k \cdot 3^{16-k} \cdot 4^k}{{}^{16}C_{k-1} \cdot 3^{17-k} \cdot 4^{k-1}}$$

$$= \frac{{}^{16}C_k}{{}^{16}C_{k-1}} \cdot \frac{4}{3}$$

$$= \frac{16\times15\times\cdots\times(16-k+1)}{1\times2\times\cdots\times k} \times \frac{1\times2\times\cdots\times(k-1)}{16\times15\times\cdots\times(16-(k-1)+1)} \cdot \frac{4}{3}$$

$$= \frac{17-k}{k} \cdot \frac{4}{3}$$

For $T_{k+1} > T_k$, $\frac{T_{k+1}}{T_k} > 1$

$\therefore \frac{4(17-k)}{3k} > 1$ (4)

$68 - 4k > 3k$

$68 > 7k$

$k < 9.7 \quad \therefore k = 9$

$\therefore$ greatest coeff. is ${}^{16}C_9 \cdot 3^7 \cdot 4^9$

(c) $v^2 = 16x - 4x^2 + 20$

(i) $\frac{1}{2}v^2 = 8x - 2x^2 + 10$

$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = 8 - 4x$ (1)

$= -4(x-2)$, since $\ddot{x}$ is in the form $\ddot{x} = -n^2X$, where $X = x - 2$, the motion is SHM.

(ii) Centre: $x = 2$ (1)

(iii) Particle is at rest when $v=0$.

$\therefore 16x - 4x^2 + 20 = 0$

$x^2 - 4x - 5 = 0$

$(x+1)(x-5) = 0$

$\therefore x = -1, \quad x = 5$

$\therefore$ amplitude $= 5 - (-1)$

$= 6\text{m}$ (2)

4. (a) $2x^3 + 3x^2 - 4 = 0$

(i) $\alpha + \beta + \gamma = -\frac{3}{2}$ (1)

(ii) $\alpha\beta\gamma = \frac{4}{2} = 2$ (1)

(iii) $\alpha^2 + \beta^2 + \gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta + \beta\gamma + \alpha\gamma)$

$= \left(-\frac{3}{2}\right)^2 - 2\times0$

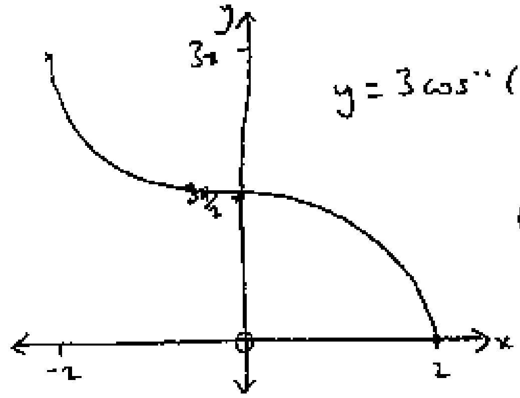
$= \frac{9}{4}$ (1)

(b) $y = 3\cos^{-1}(x/2)$

(i) D: $-2 \le x \le 2$ (2)

R: $0 \le y \le 3\pi$

(ii)



(1)

(c) Let $f(x) = x^3 + x - 1$

$f'(x) = 3x^2 + 1$

If $x_1 = 0.5$ is a close approx. to a root, then $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$ is a better approx.

$x_2 = 0.5 - \frac{f(0.5)}{f'(0.5)}$

$= 0.5 - \frac{(-0.375)}{1.75}$ (3)

$= 0.71$ (to 2 d.p.'s)

(d) $\frac{2}{\tan A + \cot A} = \sin 2A$

LHS $= \frac{2}{\tan A + \cot A} = \frac{2}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}}$

$= \frac{2\sin A\cos A}{\sin^2 A + \cos^2 A} = 2\sin A\cos A$

$= \sin 2A$

5(a) $2x - y + 5 = 0$ $y = -3x + 7$

$2x + 5 = y$

$m_1 = 2$ $m_2 = -3$

$\tan\theta = \left|\dfrac{2+3}{1+2\times(-3)}\right| = \left|\dfrac{5}{-5}\right|$

$\tan\theta = 1$ $\therefore \theta = \underline{\underline{45°}}$ (3)

(b) $N = A(1 + e^{-kt})$

(i) $t = 0,\ N = 5$

$5 = A(2)$ $A = \underline{\underline{5/2}}$ (1)

(ii) $t = 3,\ N = 80$

$80 = \frac{5}{2}(1 + e^{-3k})$

$e^{-3k} = 31$

$-3k = \ln 31$ (2)

$k = \underline{\underline{-\frac{1}{3}\ln 31}} = -1.14466$ (to 5 d.p.'s)

(iii) $560 = \frac{5}{2}(1 + e^{\frac{1}{3}\ln 31 t})$

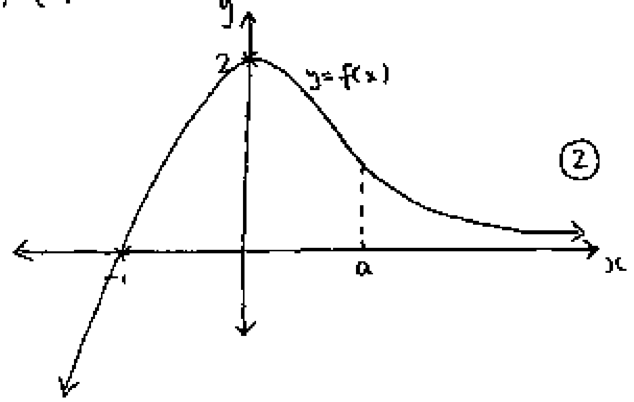
$e^{\frac{1}{3}\ln 31 t} = 223$

$\frac{1}{3}\ln 31 t = \ln 223$

$t = \underline{\underline{4.7 \text{ days}}}$ (2)

$\therefore$ all students have heard the rumour within 5 days.

(c) (i)



(ii) For $x < a$, $f(x)$ is concave down.

For $x > a$, $f(x)$ is concave up.

Hence $f(x)$ changes concavity and there is an inflexion point. (2)

6. (a) Given $\dfrac{dV}{dt} = 50$

Need to find $\dfrac{dA}{dt} = \dfrac{dA}{dr} \cdot \dfrac{dr}{dt}$

$A = 4\pi r^2$ $V = \frac{4}{3}\pi r^3$ (1)

$\dfrac{dA}{dr} = 8\pi r$ $\dfrac{dV}{dr} = 4\pi r^2$

Find $\dfrac{dr}{dt}$ from:

$\dfrac{dV}{dt} = \dfrac{dV}{dr} \cdot \dfrac{dr}{dt}$

$50 = 4\pi r^2 \cdot \dfrac{dr}{dt}$

$\dfrac{dr}{dt} = \dfrac{50}{4\pi r^2} = \dfrac{25}{2\pi r^2}$ (2)

$\therefore \dfrac{dA}{dt} = 8\pi r \cdot \dfrac{25}{2\pi r^2} = \dfrac{100}{r}$ (1)

When $r = 20$mm,

$\dfrac{dA}{dt} = \dfrac{100}{20} = \underline{\underline{5 \text{mm}^2/\text{sec}}}$ (1)

(b) (i) $x^2 = 4ay$

$y = \dfrac{x^2}{4a}$

$\dfrac{dy}{dx} = \dfrac{x}{2a}$

At $x = 2ap$, $\dfrac{dy}{dx} = \dfrac{2ap}{2a} = p$

Eqtn. of tangent at $(2ap, ap^2)$:

$y - ap^2 = p(x - 2ap)$ (2)

$y - ap^2 = px - 2ap^2$

$y - px + ap^2 = 0$ (1) (as reqd)

(ii) Tangent at Q:

$y - qx + aq^2 = 0$ (2)

Point of int:

(1) - (2):

$-px + qx + ap^2 - aq^2 = 0$

$x(q - p) = a(q^2 - p^2)$

6 (a) (ii) (cont)

$x = a(q+p)$

$y - ap(q+p) + ap^2 = 0$

$y = apq + ap^2 - ap^2$

$y = apq$

$\therefore R: \ (a(q+p),\ apq)$ (2)

(iii) grad of PO $= \dfrac{ap^2 - 0}{2ap - 0}$

$= p/2$

grad. of QO $= \dfrac{aq^2 - 0}{2aq - 0}$

$= q/2$ (1)

PO and QO are perp.,

$\therefore \dfrac{p}{2} \cdot \dfrac{q}{2} = -1$ (1)

$\therefore pq = -4$ (as reqd.)

(iv) $x = a(p+q)$

$y = apq$, but $pq = -4$,

$\therefore y = -4a$ (as reqd.) (1)

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7. (a) (i) $\sin\alpha = \dfrac{h}{2}$

$h = 2\sin\alpha$ (1)

(ii) Δ's APD and AQC are similar (AA)

$\therefore \dfrac{QC}{PD} = \dfrac{AC}{AD}$ $AD = 3 + BD$, $BD = 2\cos\alpha$

$\dfrac{x}{h} = \dfrac{6}{3+2\cos\alpha}$, but $h = 2\sin\alpha$

$\therefore x = \dfrac{12\sin\alpha}{3+2\cos\alpha}$ (as reqd.) (2)

(iii) $\dfrac{dx}{d\alpha} = \dfrac{(3+2\cos\alpha)(12\cos\alpha) - 12\sin\alpha(-2\sin\alpha)}{(3+2\cos\alpha)^2}$

$= \dfrac{36\cos\alpha + 24\cos^2\alpha + 24\sin^2\alpha}{(3+2\cos\alpha)^2}$

$= \dfrac{36\cos\alpha + 24(\cos^2\alpha + \sin^2\alpha)}{(3+2\cos\alpha)^2}$

$= \dfrac{36\cos\alpha + 24}{(3+2\cos\alpha)^2}$ (2)

Turning points occur when

$36\cos\alpha + 24 = 0$

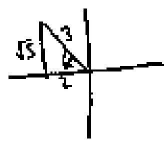
$\cos\alpha = -\dfrac{24}{36}$

$\cos\alpha = -\dfrac{2}{3}$ ($\alpha \doteqdot 2.3^c$)

$\alpha = 2.2^c$, $\dfrac{dx}{d\alpha} = 0.8467\ldots > 0$

$\alpha = 2.4^c$, $\dfrac{dx}{d\alpha} = -1.0945\ldots < 0$ (1)

$\therefore$ max. turning pt at $\alpha = 2.3^c$



$\cos\alpha = -\dfrac{2}{3}$

$\sin\alpha = \dfrac{\sqrt{5}}{3}$

$\therefore$ max. value of $x = \dfrac{12 \times \frac{\sqrt{5}}{3}}{3 + 2(-\frac{2}{3})}$

$= \dfrac{4\sqrt{5}}{5/3} = \dfrac{12\sqrt{5}}{5}$ (1)

(b) Horizontal

$\ddot{x} = 0$

$\dot{x} = C$, when $t = 0$, $\dot{x} = 25\cos\alpha$

$\therefore \dot{x} = 25\cos\alpha$

$x = 25t\cos\alpha + d$, when $t = 0, x = 0$

$\therefore x = 25t\cos\alpha$ (1)

$t = \dfrac{x}{25\cos\alpha}$ (*)

7(b) (cont.)

Vertical

$\ddot{y} = -10$

$\dot{y} = -10t + e$, when $t = 0$, $\dot{y} = 25\sin\alpha$

$\therefore \dot{y} = -10t + 25\sin\alpha$

$y = -5t^2 + 25t\sin\alpha + f$,

when $t = 0$, $y = 2$

$\therefore y = -5t^2 + 25t\sin\alpha + 2$ (1)

Subs. ⊛ into y:

$y = -5\left(\frac{x}{25\cos\alpha}\right)^2 + 25\left(\frac{x}{25\cos\alpha}\right)\cdot\sin\alpha + 2$

$= \frac{-x^2}{125}\sec^2\alpha + x\tan\alpha + 2$ (1)

When $x = 20$, $y = 15$

$15 = \frac{-400}{125}\sec^2\alpha + 20\tan\alpha + 2$

$13 = \frac{-16}{5}(1 + \tan^2\alpha) + 20\tan\alpha$

$65 = -16 - 16\tan^2\alpha + 100\tan\alpha$

$16\tan^2\alpha - 100\tan\alpha + 81 = 0$ (1)

$\tan\alpha = \frac{100 \pm \sqrt{100^2 - 4\times16\times81}}{32}$

$\tan\alpha = 0.956\ldots$, or $5.29\ldots$

$\alpha = 44°$ or $79°$

$\therefore \underline{\underline{44° \leq \alpha \leq 79°}}$ (1)